Chapter 9: STATICS AND TORQUE

# 9.2 THE SECOND CONDITION FOR EQUILIBRIUM

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| 1. | *(a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?* |
| Solution | (a)  (b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force’s application from the hinges. |
| 2. | *When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.* |
| Solution | (a)  (b) |
| 3. | *Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.* |
| Solution |  |
| 4. | *Use the second condition for equilibrium () to calculate in Example 9.1, employing any data given or solved for in part (a) of the example.* |
| Solution |  |
| 5. | *Repeat the seesaw problem in Example 9.1 with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.* |
| Solution | Given  Find  (a) Since children are balancing:  So, solving for gives:    (b) Since the children are not moving:    so that |
| 9.3 STABILITY | |
| 6. | *Suppose a horse leans against a wall as in Figure 9.32. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal and opposite to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.* |
| Solution | There are four forces acting on the horse and rider: **N** (acting straight up the ground), **w** (acting straight down from the center of mass), **f**(acting horizontally to the left, at the ground to prevent the horse from slipping), and  (acting to the right). Since nothing is moving, the two conditions for equilibrium apply:  The first condition leads to two equations (one for each direction):    The torque equation (taking torque about the center of gravity, where CCW is positive) gives:  The first two equations give:  Substituting into the third equation gives:    So, the force on the wall is : |
| 7. | *Two children of mass 20 kg and 30 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?* |
| Solution |  |
| 8. | *(a) Calculate the magnitude and direction of the force on each foot of the horse in Figure 9.32 (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.* |
| Solution | (a) With two feet on ground, each foot takes half of the total friction and normal forces, calculated as in the exercise above. Each foot feels a frictional force , directed towards the wall, and a normal force , directed upwards. The total force is then, at  .  (b) |
| 9. | *A person carries a plank of wood 2 m long with one hand pushing down on it at one end with a force  and the other hand holding it up at 50 cm from the end of the plank with force . If the plank has a mass of 20 kg and its center of gravity is at the middle of the plank, what are the forces  and ?* |
| Solution | Pick the pivot at the point where is applied. Then |
| 10. | *A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in Figure 9.33. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.* |
| Solution | Take the pivot point to be at the base of the wall and ignore the thickness of the wall. The forces acting are  (the ten braces), the weight of the wall , and the normal force. Using the second condition for equilibrium,    The force exerted by the wind is:  Thus, |
| 11. | *(a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in Figure 9.34? (b) What is the ratio of this force to the chicken’s weight? (c) Does this support the contention that the chicken has a relatively stable construction?* |
| Solution | (a)  There must be friction, *f*, between the ground and the chicken’s foot.    (pivot at foot on ground)    (b)  (c) Yes , the chicken is relatively stable. It is able to adjust its center of gravity and maintain its equilibrium position with relatively little effort. |
| 12. | *Suppose the weight of the drawbridge in Figure 9.35 is supported entirely by its hinges and the opposite shore, so that its cables are slack. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances? The mass of the bridge is 2500 kg.* |
| Solution | (a)  (b) |
| 13. | *Suppose a 900-kg car is on the bridge in Figure 9.34 with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.* |
| Solution | and    (a)  (b) |
| 14. | *A sandwich board advertising sign is constructed as shown in Figure 9.36. The sign’s mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?* |
| Solution | (a) Looking at Figure 9.36, there are three forces acting on the entire sandwich board system: , acting down at the center of mass of the system, acting up at the ground for EACH of the legs. The tension and the hinge exert internal forces, and therefore cancel when considering the entire sandwich board. Using the first condition for equilibrium gives: .  The normal forces are equal, due to symmetry, and the mass is given, so we can determine the normal forces:  Now, we can determine the tension in the chain and the force due to the hinge by using the one side of the sandwich board:      The system is in equilibrium, so the two conditions for equilibrium hold:  This gives three equations:  (pivot at hinge)    To solve for the tension, use the third equation:    Therefore, substituting in the values gives:  (b) To determine the force of the hinge, and the angle at which it acts, start with the second equation, remembering that  Now, the first equation says:  so  cannot be zero, but rather , giving a force of |
| 15. | *(a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in Figure 9.36 in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?* |
| Solution | (a) If the chain breaks , .This implies that  (b) Force exerted by the hinge: |
| 16. | *A gymnast is attempting to perform splits. From the information given in Figure 9.37, calculate the magnitude and direction of the force exerted on each foot by the floor.* |
| Solution | The cheerleader’s hips and legs stiffen enough to support her weight. A free body diagram at the hips will give two leg forces acting up in the direction of her legs and a weight force vertically downward. The first condition of equilibrium says:    Therefore, the floor exerts a force of  upward along each leg |

# 9.4 APPLICATIONS OF STATICS, INCLUDING PROBLEM-SOLVING STRATEGIES

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| 17. | *To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?* |
| Solution | The forces involved are the weight of the man , the weight of the ladder , the normal force of the ground on the ladder bottom , the normal force of the gutter on the ladder top , and friction between the ground and ladder bottom . The condition of no net force horizontally leads to , where  is the angle between the ladder and the ground: . The condition of no net force vertically leads to , which combines with the previous condition to give . The condition of no torque about the ladder bottom leads to , which combines with the first condition to give .  Combining these last two conditions, we can solve for :    We can use this value to solve for  and :    The magnitude of the force at the top is , and the force at the bottom is the sum of friction and the normal force, with a magnitude of . |
| 18. | *In Figure 9.22, the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in Figure 9.20, show that the second condition for equilibrium  is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.* |
| Solution | (a) Pick the pivot at the point where the left hand exerts :    (b)  (c) Pick any pivot point and use the known forces , , and , and their lever arms from the chosen pivot, then calculate the torques to show they add to zero. |
| 9.5 SIMPLE MACHINES | |
| 19. | *What is the mechanical advantage of a nail puller—similar to the one shown in Figure 9.24—where you exert a force from the pivot and the nail is*  *on the other side? What minimum force must you exert to apply a force of to the nail?* |
| Solution | (a)  (b) |
| 20. | *Suppose you needed to raise a 250-kg mower a distance of 6.0 cm above the ground to change a tire. If you had a 2.0-m long lever, where would you place the fulcrum if your force was limited to 300 N?* |
| Solution | Assume the lever exerts its force straight up at the center of gravity of the mower. Use the point where the fulcrum is placed as the pivot. |
| 21. | *(a) What is the mechanical advantage of a wheelbarrow, such as the one in Figure 9.25, if the center of gravity of the wheelbarrow and its load has a perpendicular lever arm of 5.50 cm, while the hands have a perpendicular lever arm of 1.02 m? (b) What upward force should you exert to support the wheelbarrow and its load if their combined mass is 55.0 kg? (c) What force does the wheel exert on the ground?* |
| Solution | (a)  (b)  (c) (see Figure 9.25 in text) |
| 22. | *A typical car has an axle with radius driving a tire with a radius of . What is its mechanical advantage assuming the very simplified model in Figure 9.26(b)?* |
| Solution |  |
| 23. | *What force does the nail puller in Exercise 9.19 exert on the supporting surface? The nail puller has a mass of 2.10 kg.* |
| Solution |  |
| 24. | *If you used an ideal pulley of the type shown in Figure 9.27(a) to support a car engine of mass , (a) What would be the tension in the rope? (b) What force must the ceiling supply, assuming you pull straight down on the rope? Neglect the pulley system’s mass.* |
| Solution | (a)  (b) |
| 25. | *Repeat Exercise 9.24 for the pulley shown in Figure 9.27(c), assuming you pull straight up on the rope. The pulley system’s mass is* *.* |
| Solution | (a)  (b) |
| 9.6 FORCES AND TORQUES IN MUSCLES AND JOINTS | |
| 26. | *Verify that the force in the elbow joint in Example 9.4 is 407 N, as stated in the text.* |
| Solution |  |
| 27. | *Two muscles in the back of the leg pull on the Achilles tendon as shown in Figure 9.38. What total force do they exert?* |
| Solution |  |
| 28. | *The upper leg muscle (quadriceps) exerts a force of 1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in Figure 9.39. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).* |
| Solution |  |
| 29. | *A device for exercising the upper leg muscle is shown in Figure 9.40, together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in Applications of Statics, Including Problem-Solving Strategies.* |
| Solution | Constant speed means  From the free body diagram of the weight:    From the net torque about the pivot: |
| 30. | *A person working at a drafting board may hold her head as shown in Figure 9.41, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae  to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.* |
| Solution |  |
| 31. | *We analyzed the biceps muscle example with the angle between forearm and upper arm set at* *. Using the same numbers as in Example 9.4, find the force exerted by the biceps muscle when the angle is*  *and the forearm is in a downward position.* |
| Solution | The answer is the same as in Example 9.4 since torque is  All angles are the same so the sine functions cancel.    Canceling out the sine functions gives the same answer as Example 9.4: |
| 32. | *Even when the head is held erect, as in Figure 9.42, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?* |
| Solution | (a)  (b) |
| 33. | *A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in Figure 9.43. (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.* |
| Solution | (a)  (b) |
| 34. | *A father lifts his child as shown in Figure 9.44. What force should the upper leg muscle exert to lift the child at a constant speed?* |
| Solution |  |
| 35. | *Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in Figure 9.45, is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the teeth on the bullet. (b) Calculate the force on the joint.* |
| Solution | (a)  (b)  Based upon the reaction force against the lower jaw as shown in Figure 9.45. |
| 36. | ***Integrated Concepts*** *Suppose we replace the 4.0-kg book in Exercise 9.31 of the biceps muscle with an elastic exercise rope that obeys Hooke’s Law. Assume its force constant* *. (a) How much is the rope stretched (past equilibrium) to provide the same force  as in this example? Assume the rope is held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of*  *with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.* |
| Solution | (a)  (b) Following Example 9.4, The condition for no net torque about the elbow joint leads to . |
| 37. | *(a) What force should the woman in Figure 9.46 exert on the floor with each hand to do a push-up? Assume that she moves up at a constant speed. (b) The triceps muscle at the back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight. (c) How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?* |
| Solution | (a) Each hand experiences an equal reaction force  , so the condition for no net torque about the feet is:    (b)    (c)  (d) |
| 38. | *You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother’s birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could you have done to ensure that the tree does not uproot easily?* |
| Solution | (a)  (b)  (c) The soil at the base of the tree could be compressed, and a stake attached to the tree would help supply a counter torque as would the compressed soil. |
| 39. | ***Unreasonable Results*** *Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?* |
| Solution | (a)  (b) The seesaw is 3.0 m long, and thus there is only 1.50 m of board on the other side of the pivot. The second child is off the board.  (c) The position of the first child must be shortened, i.e., brought closer to the pivot. |

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| Test Prep For AP® Courses | |
| 1. | *Which of the following is not an example of an object undergoing a torque?*   1. A car is rounding a bend at a constant speed. 2. A merry-go-round increases from rest to a constant rotational speed. 3. A pendulum swings back and forth. 4. A bowling ball rolls down a bowling alley. |
| Solution | (a) |
| 2. | *Five forces of equal magnitude, labeled* ***A****-****E****, are applied to the object shown below. If the object is anchored at point* ***P****, which force will provide the greatest torque?*     1. Force ***A*** 2. Force ***B*** 3. Force ***C*** 4. Force ***D*** 5. Force ***E*** |
| Solution | (d) |
| 3. | *Using the concept of torque, explain why a traffic cone placed on its base is in stable equilibrium, while a traffic cone placed on its tip is in unstable equilibrium.* |
| Solution | Both objects are in equilibrium. However, they will respond differently if a force is applied to their sides. If the cone placed on its base is displaced to the side, its center of gravity will remain over its base and it will return to its original position. When the traffic cone placed on its tip is displaced to the side, its center of gravity will drift from its base, causing a torque that will accelerate it to the ground. |
| 4. | *A child sits on the end of a playground see-saw. Which of the following values is the most appropriate estimate of the torque created by the child?*   1. 6 N·m 2. 60 N·m 3. 600 N·m 4. 6000 N·m |
| Solution | (c) |
| 5. | *A group of students is stacking a set of identical books, each one overhanging the one below it by 1 inch. They would like to estimate how many books they could place on top of each other before the stack tipped. What information below would they need to know to make this calculation?*    I. The mass of each book II. The width of each book III. The depth of each book   1. I only 2. I and II only 3. I and III only 4. II only 5. I, II, and III |
| Solution | (d) |
| 6. | *A 10 N board of uniform density is 5 meters long. It is supported on the left by a string bearing a 3 N upward force. In order to prevent the string from breaking, a person must place an upward force of 7 N at a position along the bottom surface of the board. At what distance from its left edge would they need to place this force in order for the board to be in static equilibrium?*   1. m 2. m 3. m 4. m 5. 5 m |
| Solution | (c) |
| 7. | *A bridge is supported by two piers located 20 meters apart. Both the left and right piers provide an upward force on the bridge, labeled* ***F****L and* ***F****R respectively.*   1. *If a 1000 kg car comes to rest at a point 5 meters from the left pier, how much force will the bridge provide to the left and right piers?* 2. *How will* ***F****L and* ***F****R change as the car drives to the right side of the bridge?* |
| Solution | 1. **F**L = 7350 N, **F**R = 2450 N 2. As the car moves to the right side of the bridge, **F**L will decrease and **F**R will increase. (At exactly halfway across the bridge, **F**L and **F**R will both be 4900 N.) |
| 8. | *An object of unknown mass is provided to a student. Without using a scale, design an experimental procedure detailing how the magnitude of this mass could be experimentally found. Your explanation must include the concept of torque and all steps should be provided in an orderly sequence. You may include a labeled diagram of your setup to help in your description. Include enough detail so that another student could carry out your procedure.* |
| Solution | A board can be hung from a string. On the board, a known mass can be placed at a specified distance on one end with the unknown mass placed at the opposite end. The unknown mass should be placed such that the board balances. When this occurs, the torque provided by the known mass will be equivalent to the torque provided by the unknown mass. Measure the distance to both masses. Use those distances along with the known mass in equation 9.31 to find the unknown mass. |
| 9. | *As a young student, you likely learned that simple machines are capable of increasing the ability to lift and move objects. Now, as an educated AP Physics student, you are aware that this capability is governed by the relationship between force and torque.*  *In the space below, explain why torque is integral to the increase in force created by a simple machine. You may use an example or diagram to assist in your explanation. Be sure to cite the mechanical advantage in your explanation as well.* |
| Solution | The student should mention that the guiding principle behind simple machines is the second condition of equilibrium. Though the torque leaving a machine must be equivalent to torque entering a machine, the same requirement does not exist for forces. As a result, by decreasing the lever arm to the existing force, the size of the existing force will be increased. The mechanical advantage will be equivalent to the ratio of the forces exiting and entering the machine. |
| 10. | *Figure 9.24(a) shows a wheelbarrow being lifted by an applied force* ***F****i. If the wheelbarrow is filled with twenty bricks massing 3 kg each, estimate the value of the applied force* ***F****i. Provide an explanation behind the total weight* ***w*** *and any reasoning toward your final answer. Additionally, provide a range of values over which you feel the force could exist.* |
| Solution | Students should determine the weight ***w*** by multiplying the total mass by the gravitational acceleration, 9.8 m/s2. The input force should be determined by estimating distances ***lo*** and ***li*** as shown in Figure 9.24 and using the concept of mechanical advantage to discover an answer. The range of appropriate values should be reasonable and should not have an upper or lower value greater than 50% of the estimated force provided. |
| 11. | *When you use your hand to raise a 20 lb dumbbell in a curling motion, the force on your bicep muscle is not equal to 20 lb.*   1. *Compare the size of the force placed on your bicep muscle to the force of the 20 lb dumbbell lifted by your hand. Using the concept of torque, which force is greater and explain why the two forces are not identical.* 2. *Does the force placed on your bicep muscle change as you curl the weight closer toward your body? (In other words, is the force on your muscle different when your forearm is 90° to your upper arm than when it is 45° to your upper arm?) Explain your answer using torque.* |
| Solution | 1. The force placed on your bicep muscle will be greater than the force placed on the dumbbell. The bicep muscle is closer to your elbow than the downward force placed on your hand from the dumbbell. Because the elbow is the pivot point of the system, this results in a decreased lever arm for the bicep. As a result, the force on the bicep must be greater than that placed on the dumbbell. (How much greater? The ratio between the bicep and dumbbell forces is equal to the inverted ratio of their distances from the elbow. If the dumbbell is ten times further from the elbow than the bicep, the force on the bicep will be 200 pounds!) 2. The force placed on your bicep muscle will decrease. As the forearm lifts the dumbbell, it will get closer to the elbow. As a result, the torque placed on the arm from the weight will decrease and the countering torque created by the bicep muscle will do so as well. |

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